OPTIMIZATION OF WELL LOCATIONS AND PUMPING RATES IN COASTAL AQUIFERS

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Abstract. A multi-objective optimization approach is used to determine pumping rates and well locations to prevent saltwater intrusion, while satisfying desired extraction rates in coastal aquifers. The problem is formulated to maximize pumping rates while minimizing the distance between stagnation points of extraction wells and the reference coastline location, such that the wells are placed as closely as possible to the coastline. The efficiency of the optimization process is improved by solving the problem through a sub-domain perturbation approach. The sequential use of multi-objective criteria, with pre-selected weights, gives the model the capability to achieve two objectives simultaneously (these results are not provided in this proceeding). This approach provides cost effective solutions to an important management problem in coastal aquifers.

INTRODUCTION

The two important objectives that are associated with the management of groundwater extraction in coastal aquifers is the maximization of the water supply and the minimization of the cost of this supply, while avoiding saltwater intrusion at all times. In coastal areas, where the groundwater is the major or the only source of freshwater, achieving these goals is of great interest.

The proposed approach can be effectively implemented to identify optimal water extraction conditions in surficial aquifers in Brunswick or Savanah, Ga.

BACKGROUND AND RELATED WORK

In the multi-objective control and management problem, associated with saltwater intrusion in coastal aquifers, various challenging questions should be answered. For example, answers to the following questions are of interest to managers: What is the optimal pumping rate for each existing well in a coastal aquifer that does not allow saltwater intrusion? From a more practical point, where should these wells be placed and what should their pumping rates be to avoid saltwater intrusion? How close can these wells be placed to the coast, and still avoid saltwater intrusion? The mathematical models presented in this paper can be used to help answer these questions.

The use of optimization approach in the solution of saltwater intrusion problems are relatively recent and few (Shamir et al., 1984; Willis and Finney, 1988; Finney et al., 1992; Hallaji and Yazicigil, 1996; Emch and Yeh 1998; Das and Datta, 1999a, 1999b; Cheng et al., 2000). Cheng et al. (2000) used the analytical solution of the sharp-interface saltwater intrusion model in their model to solve for pumping rates for an existing multiple well extraction scenario in a coastal aquifer with the Structured Messy Genetic Algorithm (SMGA).

METHODS

Following Cheng et al. (2000), the analytical solution of the steady state sharp-interface saltwater intrusion model is also used in this study. This solution is based on the single-potential formulation of Strack (1976). While they used the well location to detect well intrusion, we have used the well stagnation point location instead. This is because stagnation point intrusion always occurs first and if well stagnation point is intruded, then the control on saltwater intrusion is already lost. The computational process is also extended to include the determination of the optimal well location as well as the pumping rate for extraction wells. The constraint on intrusion of the stagnation point by the saltwater wedge is computed by allowing a finite distance between the positions of these two locations. In this study, we show that this approach can improve the results of the optimized solution significantly.

The formulation of the optimization problem is of great importance to reduce unnecessary simulations, before the search begins. This concept is well incorporated in this study using the Progressive Genetic Algorithm (PGA), which uses the iterative sub-domain solution introduced by Aral and Guan, (1997) and Guan and Aral, (1999a, 1999b). The use of the sub-domain perturbation method makes it possible to optimize the pumping rates and the locations of multiple wells simultaneously when these unknown variables are continuous.

FORMULATION OF SALTWATER INTRUSION PROBLEM

In most cases, one of the objectives in the optimal solution of a saltwater intrusion problem is maximizing the pumping rate. The saltwater intrusion problem becomes unique, in a way, if the extraction wells are forced to be placed as closely as possible to the coast, because of the proximity of the well field region to the coast. These situations, in practice, may cause saltwater intrusion problems. If the extraction wells can be placed further away from the coastline, all the extraction wells may be allowed to pump higher rates of fresh water with no limitations. In order to integrate this constraint into the saltwater intrusion problem, we have introduced a second objective. That is, all extraction wells should be placed as close to the coastline as possible. This, in turn, restricts the first objective, that is maximizing the pumping rate. In this work we use the single scalar objective function approach for simplicity. The two objectives of the optimization problem are given as follows:

The first objective: Maximizing the pumping rate.

$$Max \sum_{i=1}^{n} Q_i \qquad (1)$$

The second objective: Minimizing the summation of each distance between the stagnation point of a well and the reference coastline location.

$$Min \sum_{i=1}^{n} (x_c^i - x_{ref}) \qquad (2)$$

Equation (2) is modified to convert the second objective to be a maximization problem,

$$Max\left(- \sum_{i=1}^{n} (x_c^i - x_{ref}) \right) \qquad (3)$$

The combined and normalized objective function can be written as,

$$f\left(Q, x_{ref}, x_c^i\right) = \sum_{i=1}^{n} \left(a \frac{Q_i}{Q_i^{\max}} + b\left(\frac{x_{ref}}{x_c^i} - 1 \right) \right) \qquad (4)$$

Subject to the following conditions,

$$x_{toe}^i(Q, X, Y) < x_c^i(Q, X, Y) \qquad (5)$$

$$\begin{aligned} &Q_i^{\min} < Q_i < Q_i^{\max}, \\ &x_i^{\min} < x_i < x_i^{\max}, \\ &y_i^{\min} < y_i < y_i^{\max} \quad i = 1, \ldots, n \end{aligned} \qquad (6)$$

where Q_i is the pumping rate of well *i*, x_c^i is the stagnation point associated with the pumping well Q_i, x_{ref} is the coastline, x_{toe}^i is the toe location (the leading edge of saltwater), and a and b are the multi-objective function weighting parameters. X and Y are vectors of well locations. The independent variable vectors, Q, X, Y, will be converted to the perturbation of each to solve for the trace of the problem ($\Delta Q, \Delta X, \Delta Y$) rather than Q, X, Y themselves. The proposed model perturbs the well location and pumping rate within the sub-domain, and GA is applied to simultaneously find the optimal well location and pumping rate in a pre-defined solution domain. Due to limited space, the description of modified sub-domain optimization problem formulation and the design of PGA are omitted in this paper. The details of PGA can be found in Aral and Guan (1997, 1999).

APPLICATION

One of the practical applications of the proposed model is the solution of the saltwater intrusion problem for fixed well locations. To test the current model for the case of fixed well location, a comparison between Cheng et al.'s work (2000) and the model developed is presented for two cases (i.e., pumping rates only).

The first case is the Case 1 of the example problems presented in Cheng et al. (2000) which consists of eight wells. The second is the Case 3 of the same work which contains seven wells. In their work, the authors initially started with a total of fifteen wells and eventually screened down to eight and seven. Here, we compare our results with their results for only eight and seven wells cases. The pumping rates of all wells are initially set to the minimum pumping rate, 150 m^3/d, as suggested by Cheng et al. (2000). While these solutions were obtained by Cheng et al. (2000) on a Pentium 450-MHz microcomputer using about 6 hours of CPU time, we have used less than 30 minutes of simulation time on a compatible machine to obtain our results. Both results yielded higher pumping rates than those of Cheng et al (Table 1). One of the reasons for this may be that the independent variable in this study is selected as a continuous perturbation, while the independent variable of Cheng et al's is the discretized pumping rate. The continuous variable approach may provide a more flexible design and lead to a better result. The detailed well-by-well comparison is provided in Table 1. Figure 1 (a) and (b) describe the optimized well locations, stagnation points, and saltwater wedge developed in this study of Case 1 and 3 problems.

In our solution, as an extension, the existing wells can be moved to new locations to further increase the total pumping rate while avoiding saltwater intrusion. This, for practical purposes, implies that the old wells are to be abandoned and new wells must be placed at the new locations with new pumping rates. The model developed is a very useful tool to analyze such cases. Based on the previous results, another simulation is conducted to see if optimization of the well locations can improve the solution. The proposed model is capable of assigning a moving well or wells to any number of wells (i.e., none, some or all of the wells) so that it seeks for new locations through optimization. The simulation was conducted by moving only one well which is the first well of Case 3 in Table 1. In this case, the total pumping rate increased to 4642.8 m^3/d from 3897 m^3/d (Figure 1(c) and Table 1). This suggests that optimization of the well locations together with the pumping rates improves the results significantly.

CONCLUSIONS

The problems we have tested and presented here have shown that the proposed mathematical formulation have produced optimal solution in an efficient manner. The formulation of the optimization problem combined with GA is straightforward and can be applied to the homogenous coastal aquifers under steady state freshwater flow. While the previous work often focuses on indirect methods or pumping rate optimization only, in this paper we present a new formulation in which the perturbations of pumping rates and well locations are used as continuous independent variables explicitly. This approach produces the optimal solutions not only for the fixed-well cases, but it also handles the optimization of well location cases as well. The advantage is a significant reduction in model runs which consequently reduces the computational time and cost. The constraint for detecting the well intrusion uses the stagnation point of a well instead of well location. This is a more limiting constraint for the well-intrusion detection. Physically, it is the well stagnation point that is intruded first by the saltwater wedge than the well location itself. This in turn may have improved the optimal solution we have obtained when compared to the results obtained earlier.

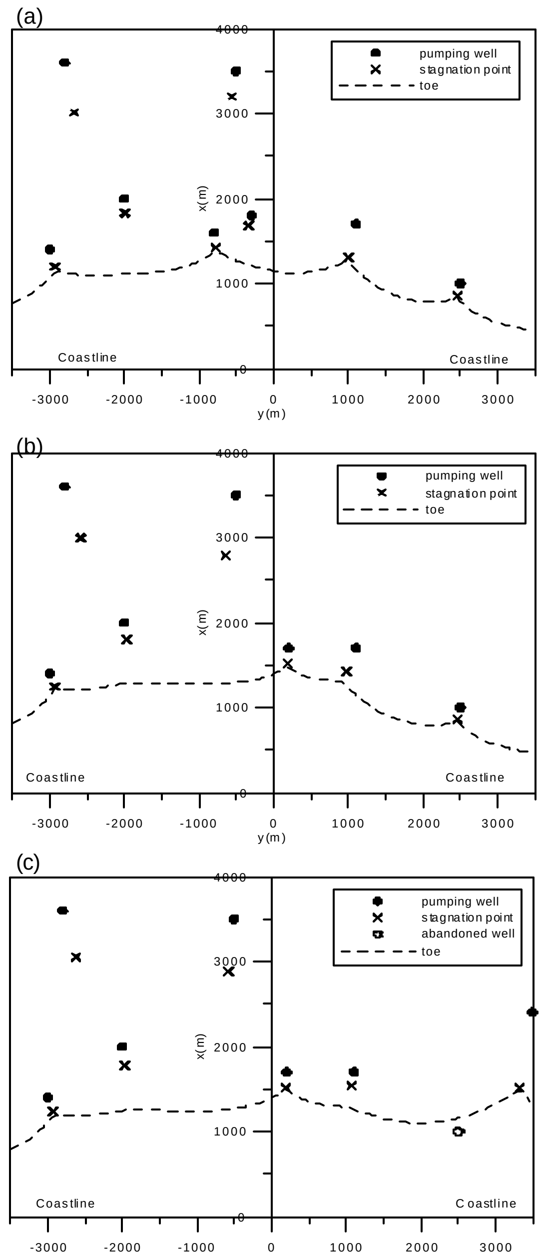

Figure 1. Comparison of results between Cheng et al. (2000) and MOGA application; (a) Case 1 results; (b) Case 3 results. (c) MOGA results obtained after moving one well given in Case 3.

Table 1. The comparison of results with Cheng et al. (2000) fixed well optimization example

Starting Points (fixed)			Optimal Pumping Rate	
Q, m³/d	***x*, m**	**y, m**	**Q, m³/d**	
Case 1 (eight wells) fixed-well				
			Cheng et al.'s	This work
$Q_1 = 150$	$x_1 = 1000$	$y_1 = 2500$	$Q_1 = 255$	$Q_1 = 221.7$
$Q_2 = 150$	$x_2 = 1700$	$y_2 = 1100$	$Q_2 = 402$	$Q_2 = 579.8$
$Q_3 = 150$	$x_3 = 1800$	$y_3 = -300$	$Q_3 = 158$	$Q_3 = 154.4$
$Q_4 = 150$	$x_4 = 3500$	$y_4 = -500$	$Q_4 = 728$	$Q_4 = 733.2$
$Q_5 = 150$	$x_5 = 1600$	$y_5 = -800$	$Q_5 = 150$	$Q_5 = 151.1$
$Q_6 = 150$	$x_6 = 3600$	$y_6 = -2800$	$Q_6 = 1500$	$Q_6 = 1402.9$
$Q_7 = 150$	$x_7 = 1400$	$y_7 = -3000$	$Q_7 = 185$	$Q_7 = 215.9$
$Q_8 = 150$	$x_8 = 2000$	$y_8 = -2000$	$Q_8 = 232$	$Q_8 = 178.4$
			$Q_{total} = 3610$	$Q_{total} = 3637.4$
Case 3 (seven wells) fixed-well				
			Cheng et al.'s	This work
$Q_1 = 150$	$x_1 = 1000$	$y_1 = 2500$	$Q_1 = 201$	$Q_1 = 198.1$
$Q_2 = 150$	$x_2 = 1700$	$y_2 = 1100$	$Q_2 = 351$	$Q_2 = 380.0$
$Q_3 = 150$	$x_3 = 1700$	$y_3 = 200$	$Q_3 = 150$	$Q_3 = 150.1$
$Q_4 = 150$	$x_4 = 3500$	$y_4 = -500$	$Q_4 = 1497$	$Q_4 = 1462.0$
$Q_5 = 150$	$x_5 = 2000$	$y_5 = -2000$	$Q_5 = 155$	$Q_5 = 150.0$
$Q_6 = 150$	$x_6 = 3600$	$y_6 = -2800$	$Q_6 = 1387$	$Q_6 = 1406.6$
$Q_7 = 150$	$x_7 = 1400$	$y_7 = -3000$	$Q_7 = 150$	$Q_7 = 150.2$
			$Q_{total} = 3891$	$Q_{total} = 3897.0$

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